Feynman units

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Feynman units!?

- It is a clickbait, there is no such unit system.
- But I would like to go over constants, units and notations of electrodynamics
- and introduce what Feynman used in this *Lectures on Physics (FLP)* to you
- to summarize
 - Among ε_0 , μ_0 , c, we only need two of them
 - Feynman used ε_0 and c and I think this makes more sense
 - B in Gaussian units is different quantity than B in SI

Microscopic Maxwell's equations

$$abla imes \boldsymbol{E} + rac{\partial \boldsymbol{B}}{\partial t} = 0 \qquad \qquad \nabla \cdot \boldsymbol{E} = \rho/\varepsilon_0$$

$$abla imes oldsymbol{B} - \mu_0 arepsilon_0 \, rac{\partial oldsymbol{E}}{\partial t} = \mu_0 \, oldsymbol{j} \qquad
abla \cdot oldsymbol{B} = 0$$

We have two constants: $\varepsilon_0 \quad \mu_0$.

Two constants: $\varepsilon_0 \quad \mu_0$

 ε_0 and μ_0 relates charge and current to mechanical force, respectively.

$$F = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q_1 Q_2}{r^2} \qquad F = \frac{\mu_0}{2\pi} \cdot \frac{I_1 I_2}{r^2}$$

Since current is flow of charge, ε_0 and μ_0 cannot be independent. Indeed, they have following relationship:

$$\varepsilon_0 \mu_0 = \frac{1}{c^2}$$
 or $\mu_0 = \frac{1}{\varepsilon_0 c^2}$

While c (the speed of light) does not depend on how we chose the unit of charge

How about using ε_0 and c instead of ε_0 and μ_0 ?

$$abla imes \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = 0 \qquad \qquad \nabla \cdot \mathbf{E} = \rho / \varepsilon_0$$

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- This is what Feynman used in his "Lectures on Physics"
- ▶ Notice E and B are in different dimension $([\nabla] = L^{-1}, \left[\frac{\partial}{\partial t}\right] = T^{-1})$

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- lt is cB that has the same dimension as E $([\nabla] = \begin{bmatrix} \frac{1}{c} \frac{\partial}{\partial t} \end{bmatrix} = L^{-1})$
- And coefficient of j is now $1/\varepsilon_0 c$

$$\nabla \times \boldsymbol{E} + \frac{\partial \boldsymbol{B}}{\partial t} = 0 \qquad \qquad \nabla \cdot \boldsymbol{E} = \rho/\varepsilon_0$$

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Coefficient of
$$\boldsymbol{j}$$
: $1/\varepsilon_0 c$

Recalling that $1/c = \sqrt{\varepsilon_0 \mu_0}$

$$\frac{1}{\varepsilon_0 c} = \frac{1}{\varepsilon_0} \cdot \sqrt{\varepsilon_0 \mu_0} = \sqrt{\frac{\mu_0}{\varepsilon_0}} \sim 377\Omega$$

This is impedance of free space

$$\nabla \times \boldsymbol{E} + \frac{1}{c} \frac{\partial c \boldsymbol{B}}{\partial t} = 0 \qquad \nabla \cdot \boldsymbol{E} = \rho/\varepsilon_0$$

$$\nabla \times c\boldsymbol{B} - \frac{1}{c}\frac{\partial \boldsymbol{E}}{\partial t} = \frac{\boldsymbol{j}}{\varepsilon_0 c} \qquad \nabla \cdot c\boldsymbol{B} = 0$$

• We have two constants, ε_0 and c

- $\triangleright \varepsilon_0$ is for charge. c is for electromagnetic field
- Coefficient of j is impedance of free space, 377Ω

$$\nabla \times \boldsymbol{E} + \frac{1}{c} \frac{\partial c \boldsymbol{B}}{\partial t} = 0 \qquad \nabla \cdot \boldsymbol{E} = \rho/\varepsilon_0$$
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Why dimension is so important?

Dimension check is primary method to detect errors in your calculation

You cannot equate, add or subtract quantities in different dimension

Simpler dimension makes error detection easier

$u \text{ and } \boldsymbol{S}$

$$u = \frac{\varepsilon_0}{2} \left(|\boldsymbol{E}|^2 + |c\boldsymbol{B}|^2 \right)$$

$$\boldsymbol{S} = \varepsilon_0 c \left(\boldsymbol{E} \times c \boldsymbol{B} \right)$$

Note that ε_0 is capacitance per length, E and cB is voltage per length, and $1/\varepsilon_0 c$ is resistance.

Gaussian units

$$abla imes \boldsymbol{E} + rac{1}{c} rac{\partial \boldsymbol{B}}{\partial t} = 0 \qquad \qquad \nabla \cdot \boldsymbol{E} = 4\pi\rho$$

$$abla imes \mathbf{B} - rac{1}{c} rac{\partial \mathbf{E}}{\partial t} = rac{4\pi}{c} \mathbf{j} \qquad \nabla \cdot \mathbf{B} = 0$$

- This is Maxwell's equations in Gaussian units
- ▶ Notice that *E* and *B* are in the same unit (dimension) 🖒
- ▶ It has dimensionless number 4π instead of $1/\varepsilon_0$, i.e., $F = \frac{Q_1 Q_2}{r^2}$ in Gaussian
- $4\pi/c$ is still impedance of free space, but in seconds per centimeter!

Gaussian units is popular among physicists

"Unfortunately one of the results of the completely disconnected way in which electricity and magnetism have been taught in the past has been the growing acceptance of the mks over the cgs system of units. We have no special preference for centimeters over meters or of grams over kilograms. We do, however, require a system wherein the electric field E and the magnetic field Bare in the same unit."

— Melvin Schwarts, Principles of Electrodynamics, (1972)

Gaussian units is popular among physicists, but ...

"My tardy adoption of the universally accepted SI system is recognition that almost all undergraduate physics texts, as well as engineering book at all levels, employ SI units throughout. For many years Ed Purcell (1912–1997) and I had a pact to support each other in the use of Gaussian units. Now I have betrayed him!"

— John David Jackson, Classical Electrodynamics, (1998)

"For 50 years, Edward Purcell's classic textbook has introduced students to the world of electricity and magnetism. This third edition has been brought up to date and is now in SI units."

- Edward M. Purcell and David J. Morin, *Electricity and Magnetism*, (2013)

Gaussian units with ε_0

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$$abla imes \mathbf{B} - \frac{1}{c} \frac{\partial \mathbf{E}}{\partial t} = \frac{1}{\varepsilon_0 c} \mathbf{j} \qquad \nabla \cdot \mathbf{B} = 0$$

- Notice similarity to equations in SI with ε_0 and c
- Substitute $1/\varepsilon_0$ with 4π to go to Gaussian
- Substitute B with cB to go to SI
- B in SI is not the same quantity as B in Gaussian units! (cB is)

Hall coefficient

Now we see why Hall coefficient is different.

In SI, $R_H = 1/nq$,

$$E_y = R_H j_x B = \frac{1}{nq} j_x B = \frac{1}{nqc} j_x cB \qquad (SI)$$

Perform $c {m B}
ightarrow {m B}$ to go to Gaussian

$$E_y = \frac{1}{nqc} \; j_x B \qquad (Gaussian)$$

Therefore

$$R_H = \frac{1}{nqc} \qquad (Gaussian)$$

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 \blacktriangleright **E** and **cB** are independent. Electrostatics and magnetostatics are distinct

 \blacktriangleright May make sense to use μ_0 , because E is not related to B

$$\nabla \times \boldsymbol{E} + \frac{1}{c} \frac{\partial c \boldsymbol{B}}{\partial t} = 0$$
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Remove time derivatives

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• What happens if
$$c \to \infty$$
?

$$\blacktriangleright$$
 $oldsymbol{B}$ vanishes. c has to be finite for $oldsymbol{B}$ to exist

► Magnetism is relativistic effect

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Magnetism is relativistic effect

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- What happens if $c \to \infty$?
- **b B** vanishes. *c* has to be finite for **B** to exist
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ho and \boldsymbol{j}

 ρ and j includes all the charge, electrons and ions.

$$ho = \sum_i q_i \, \delta^3(\boldsymbol{r} - \boldsymbol{r}_i) \qquad \quad \boldsymbol{j} = \sum_i \boldsymbol{v}_i \, q_i \, \delta^3(\boldsymbol{r} - \boldsymbol{r}_i)$$

We introduce P and M to bridge microscopic world to macroscopic world.

$$ho =
ho^{(\mathrm{f})} -
abla \cdot oldsymbol{P} \qquad oldsymbol{j} = oldsymbol{j}^{(\mathrm{f})} + rac{\partial oldsymbol{P}}{\partial t} +
abla imes oldsymbol{M}$$

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$$abla \cdot oldsymbol{E} = rac{1}{arepsilon_0}
ho$$
 $abla imes coldsymbol{B} - rac{1}{c} rac{\partial oldsymbol{E}}{\partial t} = rac{1}{arepsilon_0 c} oldsymbol{j}$

 \blacktriangleright Insert previous page's definition and move P and M to the other side

- ▶ I wish I could use $D = E + P/\varepsilon_0$ and $H = cB M/\varepsilon_0 c$ (it's cleaner 🖒)
- We customary use $D = \varepsilon_0 E + P$ and $H = \varepsilon_0 c^2 B M$ (more units \mathbb{Q})

Feynman used $H = B - M/\varepsilon_0 c^2$ to make H to have the same units as B

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$$abla \cdot \boldsymbol{E} = rac{1}{arepsilon_0} \left(
ho^{(\mathrm{f})} -
abla \cdot \boldsymbol{P}
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 $c^2
abla imes \left(oldsymbol{B} - oldsymbol{M} / arepsilon_0 c^2
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• We customary use $D = \varepsilon_0 E + P$ and $H = \varepsilon_0 c^2 B - M$ (more units \mathbb{Q})

 \blacktriangleright Feynman used $m{H} = m{B} - m{M} / arepsilon_0 c^2$ to make $m{H}$ to have the same units as $m{B}$

Macroscopic Maxwell's equations (SI with ε_0 and c)

$$abla imes oldsymbol{E} + rac{\partial oldsymbol{B}}{\partial t} = 0 \qquad
abla \cdot oldsymbol{B} = 0$$
 $abla \cdot oldsymbol{D} =
ho^{(\mathrm{f})} \qquad oldsymbol{D} = (arepsilon_0 oldsymbol{E} + oldsymbol{P})$
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ho^{(\mathrm{f})} \qquad oldsymbol{D} = (arepsilon_0 oldsymbol{E} + oldsymbol{P})$
 $abla \cdot oldsymbol{D} = oldsymbol{j}^{(\mathrm{f})} \qquad oldsymbol{H} = (arepsilon_0 oldsymbol{C}^2 oldsymbol{B} - oldsymbol{M})$

E, B, D, H are in different units

Some people in the past thought this is cleaner, because constants are hidden

Macroscopic Maxwell's equations (Feynman)

$$\nabla \times \boldsymbol{E} + \frac{\partial \boldsymbol{B}}{\partial t} = 0 \qquad \nabla \cdot \boldsymbol{B} = 0$$
$$\nabla \cdot (\varepsilon_0 \boldsymbol{E} + \boldsymbol{P}) = \rho_{\text{other}} \qquad \boldsymbol{D}/\varepsilon_0 = (\boldsymbol{E} + \boldsymbol{P}/\varepsilon_0)$$
$$c^2 \nabla \times \left(\boldsymbol{B} - \frac{\boldsymbol{M}}{\varepsilon_0 c^2}\right) = \frac{\boldsymbol{j}_{\text{cond}}}{\varepsilon_0} + \frac{\partial}{\partial t} \left(\boldsymbol{E} - \frac{\boldsymbol{P}}{\varepsilon_0}\right) \qquad \boldsymbol{H} = \left(\boldsymbol{B} - \frac{\boldsymbol{M}}{\varepsilon_0 c^2}\right)$$

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Please read FLP Vol II Chap 36

Macroscopic Maxwell's equations (Gaussian units)

$$\nabla \times \boldsymbol{E} + \frac{1}{c} \frac{\partial \boldsymbol{B}}{\partial t} = 0 \qquad \nabla \cdot \boldsymbol{B} = 0$$
$$\nabla \cdot \boldsymbol{D} = 4\pi \rho^{(f)} \qquad \boldsymbol{D} = (\boldsymbol{E} + 4\pi \boldsymbol{P})$$
$$\nabla \times \boldsymbol{H} - \frac{1}{c} \frac{\partial \boldsymbol{D}}{\partial t} = \frac{4\pi}{c} \boldsymbol{j}^{(f)} \qquad \boldsymbol{H} = (\boldsymbol{B} - 4\pi \boldsymbol{M})$$

 $oldsymbol{E}$, $oldsymbol{B}$, $oldsymbol{D}$, $oldsymbol{H}$ are in the same unit

D and H in Gaussian are different quantities than those in SI

Macroscopic Maxwell's equations (Gaussian units with ε_0)

$$\nabla \times \boldsymbol{E} + \frac{1}{c} \frac{\partial \boldsymbol{B}}{\partial t} = 0 \qquad \nabla \cdot \boldsymbol{B} = 0$$

$$abla \cdot oldsymbol{D} =
ho^{(\mathrm{f})} / arepsilon_0 \qquad oldsymbol{D} = (oldsymbol{E} + oldsymbol{P} / arepsilon_0)$$

$$abla imes oldsymbol{H} - rac{1}{c} rac{\partial oldsymbol{D}}{\partial t} = rac{1}{arepsilon_0 c} \, oldsymbol{j}^{(\mathrm{f})} \qquad oldsymbol{H} = (oldsymbol{B} - oldsymbol{M} / arepsilon_0)$$

Substitute 4π with $1/\varepsilon_0$ to give charge a dimension

Macroscopic Maxwell's equations (Gaussian units with ε_0)

$$abla imes \boldsymbol{E} + rac{1}{c} rac{\partial \boldsymbol{B}}{\partial t} = 0 \qquad \nabla \cdot \boldsymbol{B} = 0$$

$$abla \cdot oldsymbol{E} = rac{1}{arepsilon_0} \left(
ho^{(\mathrm{f})} -
abla \cdot oldsymbol{P}
ight)$$

$$abla imes oldsymbol{B} - rac{1}{c} rac{\partial oldsymbol{E}}{\partial t} = rac{1}{arepsilon_0 c} \left(oldsymbol{j}^{(\mathrm{f})} + rac{\partial oldsymbol{P}}{\partial t} +
abla imes coldsymbol{M}
ight)$$

EM field in the left, material in the right hand side

You can go to SI with $oldsymbol{B}
ightarrow coldsymbol{B}$ and $oldsymbol{M}
ightarrow oldsymbol{M} / c$

$c \quad \varepsilon_0 \quad q_e$

"~" means measured, "=" means defined q_e is elementary charge

Gaussian

$$c \sim 2.998 \times 10^{10} \,\mathrm{cm/s}$$
 $\varepsilon_0 = \frac{1}{4\pi}$ $q_e \sim 4.803 \times 10^{-10} \,\mathrm{statC} \left(\mathrm{cm}^{3/2} \mathrm{g}^{1/2} \mathrm{s}^{-1}\right)$

SI before 2019

$$c = 299792458 \,\mathrm{m/s}$$
 $\varepsilon_0 = \frac{10^7}{4\pi \left(c/(\mathrm{m/s})\right)^2} \,\mathrm{F/m}$ $q_e \sim 1.602 \times 10^{-19} \,\mathrm{C}$

SI after 2019

 $c = 299792458 \,\mathrm{m/s}$ $\varepsilon_0 \sim 8.8854 \times 10^{-12} \,\mathrm{F/m}$ $q_e = 1.602176634 \times 10^{-19} \,\mathrm{C}$

Gaussian units cannot be accurate theory any longer, because you can't modify $1/4\pi$

Dimensions for SI quantities ([\circ] reads dimension of \circ)

$$\begin{bmatrix} \boldsymbol{E} \end{bmatrix} = \begin{bmatrix} c\boldsymbol{B} \end{bmatrix} = \begin{bmatrix} \frac{\text{Voltage}}{\text{Length}} \end{bmatrix} \quad \begin{bmatrix} \rho \end{bmatrix} = \begin{bmatrix} \frac{\text{Charge}}{\text{Length}^3} \end{bmatrix} \quad \begin{bmatrix} \boldsymbol{j} \end{bmatrix} = \begin{bmatrix} \frac{\text{Current}}{\text{Length}^2} \end{bmatrix} \quad \begin{bmatrix} \varepsilon_0 \end{bmatrix} = \begin{bmatrix} \frac{\text{Cap}}{\text{Length}} \end{bmatrix} \\ \begin{bmatrix} \nabla \cdot \boldsymbol{E} = \frac{\rho}{\varepsilon_0} \end{bmatrix} = \begin{bmatrix} \frac{\text{Voltage}}{\text{Length}^2} = \frac{\text{Charge}}{\text{Cap} \cdot \text{Length}^2} \end{bmatrix} \quad \begin{pmatrix} V = \frac{Q}{C} \end{pmatrix} \quad \begin{bmatrix} \varepsilon_0 \frac{S}{d} \end{bmatrix} = \begin{bmatrix} \text{Cap} \end{bmatrix} \\ \begin{bmatrix} \text{Time} \end{bmatrix} = \begin{bmatrix} \text{Res} \cdot \text{Cap} \end{bmatrix} \quad (\tau = RC) \quad \begin{bmatrix} 1/\varepsilon_0 c \end{bmatrix} = \begin{bmatrix} \text{Time}/\text{Cap} \end{bmatrix} = \begin{bmatrix} \text{Res} \end{bmatrix} \\ \begin{bmatrix} \nabla \times c\boldsymbol{B} - \frac{1}{c}\frac{\partial \boldsymbol{E}}{\partial t} = \frac{1}{\varepsilon_0 c}\boldsymbol{j} \end{bmatrix} = \begin{bmatrix} \frac{\text{Voltage}}{\text{Length}^2} = \text{Res} \cdot \frac{\text{Current}}{\text{Length}^2} \end{bmatrix} \quad (V = RI) \\ \begin{bmatrix} \boldsymbol{j} = \sigma \boldsymbol{E} \end{bmatrix} = \begin{bmatrix} \frac{\text{Current}}{\text{Length}^2} = \frac{1}{\text{Res} \cdot \text{Length}} \cdot \frac{\text{Voltage}}{\text{Length}} \end{bmatrix} \quad \begin{pmatrix} I = \frac{V}{R} \end{pmatrix} \end{aligned}$$

"The difficulty of science are to a large extent the difficulties of notation, the units, and all the other artificialities which are invented by man, not by nature."

- Richard P. Feynman, The Feynman Lectures on Physics

Bibliography

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