Feynman units

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Feynman units!?

- It is a clickbait, there is no such unit system.
- But I would like to go over constants, units and notations of electrodynamics
- and introduce what Feynman used in this *Lectures on Physics* (*FLP*) to you
- to summarize
  - Among ε_0, μ_0, c, we only need two of them
  - Feynman used ε_0 and c and I think this makes more sense
  - $\boldsymbol{B}$ in Gaussian units is different quantity than $\boldsymbol{B}$ in SI

Microscopic Maxwell's equations

$$\nabla \times \boldsymbol{E} + \frac{\partial \boldsymbol{B}}{\partial t} = 0 \qquad \nabla \cdot \boldsymbol{E} = \rho/\varepsilon_0$$

$$\nabla \times \boldsymbol{B} - \mu_0 \varepsilon_0 \frac{\partial \boldsymbol{E}}{\partial t} = \mu_0 \, \boldsymbol{j} \qquad \nabla \cdot \boldsymbol{B} = 0$$

We have two constants: $\varepsilon_0 \quad \mu_0$.

Two constants: ε_0 μ_0

ε_0 and μ_0 relates charge and current to mechanical force, respectively.

$$F = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q_1 Q_2}{r^2} \qquad F = \frac{\mu_0}{2\pi} \cdot \frac{I_1 I_2}{r^2}$$

Since current is flow of charge, ε_0 and μ_0 cannot be independent.
Indeed, they have following relationship:

$$\varepsilon_0 \mu_0 = \frac{1}{c^2} \quad \text{or} \quad \mu_0 = \frac{1}{\varepsilon_0 c^2}$$

While c (the speed of light) does not depend on how we chose the unit of charge

How about using ε_0 and c instead of ε_0 and μ_0?

Using ε_0 and c instead of ε_0 and μ_0

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$$\nabla \times \boldsymbol{B} - \mu_0 \varepsilon_0 \frac{\partial \boldsymbol{E}}{\partial t} = \mu_0 \boldsymbol{j} \qquad \nabla \cdot \boldsymbol{B} = 0$$

- This is what Feynman used in his "*Lectures on Physics*"
- Notice $\boldsymbol{E}$ and $\boldsymbol{B}$ are in different dimension $\left([\nabla] = \mathsf{L}^{-1},\ \left[\frac{\partial}{\partial t}\right] = \mathsf{T}^{-1}\right)$
- It is $c\boldsymbol{B}$ that has the same dimension as $\boldsymbol{E}$ $\left([\nabla] = \left[\frac{1}{c}\frac{\partial}{\partial t}\right] = \mathsf{L}^{-1}\right)$
- And coefficient of $\boldsymbol{j}$ is now $1/\varepsilon_0 c$

Using ε_0 and c instead of ε_0 and μ_0

$$\nabla \times \boldsymbol{E} + \frac{\partial \boldsymbol{B}}{\partial t} = 0 \qquad \nabla \cdot \boldsymbol{E} = \rho / \varepsilon_0$$

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Coefficient of $\boldsymbol{j}$: $1/\varepsilon_0\, c$

Recalling that $1/c = \sqrt{\varepsilon_0 \mu_0}$

$$\frac{1}{\varepsilon_0 c} \;=\; \frac{1}{\varepsilon_0} \cdot \sqrt{\varepsilon_0 \mu_0} \;=\; \sqrt{\frac{\mu_0}{\varepsilon_0}} \;\sim\; 377\Omega$$

This is impedance of free space

Now

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- We have two constants, ε_0 and c
- ε_0 is for charge. c is for electromagnetic field
- Coefficient of j is impedance of free space, 377Ω
- We treat cB as a single symbol as it has the same dimension as E

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Why dimension is so important?

Dimension check is primary method to detect errors in your calculation

You cannot equate, add or subtract quantities in different dimension

Simpler dimension makes error detection easier

u and $\boldsymbol{S}$

$$u = \frac{\varepsilon_0}{2} \left(|\boldsymbol{E}|^2 + |c\boldsymbol{B}|^2 \right)$$

$$\boldsymbol{S} = \varepsilon_0 c \left(\boldsymbol{E} \times c\boldsymbol{B} \right)$$

Note that ε_0 is capacitance per length, $\boldsymbol{E}$ and $c\boldsymbol{B}$ is voltage per length, and $1/\varepsilon_0 c$ is resistance.

Gaussian units

$$\nabla \times \boldsymbol{E} + \frac{1}{c}\frac{\partial \boldsymbol{B}}{\partial t} = 0 \qquad \nabla \cdot \boldsymbol{E} = 4\pi\rho$$

$$\nabla \times \boldsymbol{B} - \frac{1}{c}\frac{\partial \boldsymbol{E}}{\partial t} = \frac{4\pi}{c}\boldsymbol{j} \qquad \nabla \cdot \boldsymbol{B} = 0$$

- This is Maxwell's equations in Gaussian units
- Notice that $\boldsymbol{E}$ and $\boldsymbol{B}$ are in the same unit (dimension)
- It has dimensionless number 4π instead of $1/\varepsilon_0$, i.e., $F = \frac{Q_1 Q_2}{r^2}$ in Gaussian
- $4\pi/c$ is still impedance of free space, but in seconds per centimeter!

Gaussian units is popular among physicists

"Unfortunately one of the results of the completely disconnected way in which electricity and magnetism have been taught in the past has been the growing acceptance of the mks over the cgs system of units. We have no special preference for centimeters over meters or of grams over kilograms. We do, however, require a system wherein the electric field $\boldsymbol{E}$ *and the magnetic field* $\boldsymbol{B}$ *are in the same unit."*

— Melvin Schwarts, *Principles of Electrodynamics*, (1972)

Gaussian units is popular among physicists, but ...

"My tardy adoption of the universally accepted SI system is recognition that almost all undergraduate physics texts, as well as engineering book at all levels, employ SI units throughout. For many years Ed Purcell (1912–1997) and I had a pact to support each other in the use of Gaussian units. Now I have betrayed him!"

— John David Jackson, *Classical Electrodynamics*, (1998)

"For 50 years, Edward Purcell's classic textbook has introduced students to the world of electricity and magnetism. This third edition has been brought up to date and is now in SI units."

— Edward M. Purcell and David J. Morin, *Electricity and Magnetism*, (2013)

Gaussian units with ε_0

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$$\nabla \times \boldsymbol{B} - \frac{1}{c}\frac{\partial \boldsymbol{E}}{\partial t} = \frac{1}{\varepsilon_0 c}\boldsymbol{j} \qquad \nabla \cdot \boldsymbol{B} = 0$$

- Notice similarity to equations in SI with ε_0 and c
- Substitute $1/\varepsilon_0$ with 4π to go to Gaussian
- Substitute $\boldsymbol{B}$ with $c\boldsymbol{B}$ to go to SI
- $\boldsymbol{B}$ in SI is not the same quantity as $\boldsymbol{B}$ in Gaussian units! ($c\boldsymbol{B}$ is)

Hall coefficient

Now we see why Hall coefficient is different.

In SI, $R_H = 1/nq$,

$$E_y = R_H\, j_x\, B = \frac{1}{nq}\, j_x\, B = \frac{1}{nqc}\, j_x\, cB \qquad \text{(SI)}$$

Perform $c\boldsymbol{B} \to \boldsymbol{B}$ to go to Gaussian

$$E_y = \frac{1}{nqc}\, j_x\, B \qquad \text{(Gaussian)}$$

Therefore

$$R_H = \frac{1}{nqc} \qquad \text{(Gaussian)}$$

Statics

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- Remove time derivatives
- $\boldsymbol{E}$ and $c\boldsymbol{B}$ are independent. Electrostatics and magnetostatics are distinct
- May make sense to use μ_0, because $\boldsymbol{E}$ is not related to $\boldsymbol{B}$

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Non-relativistic limit

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- What happens if $c \to \infty$?
- B vanishes. c has to be finite for B to exist
- Magnetism is relativistic effect

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ρ and $\boldsymbol{j}$

ρ and $\boldsymbol{j}$ includes all the charge, electrons and ions.

$$\rho = \sum_i q_i\, \delta^3(\boldsymbol{r} - \boldsymbol{r}_i) \qquad \boldsymbol{j} = \sum_i \boldsymbol{v}_i\, q_i\, \delta^3(\boldsymbol{r} - \boldsymbol{r}_i)$$

We introduce $\boldsymbol{P}$ and $\boldsymbol{M}$ to bridge microscopic world to macroscopic world.

$$\rho = \rho^{(\mathrm{f})} - \nabla \cdot \boldsymbol{P} \qquad \boldsymbol{j} = \boldsymbol{j}^{(\mathrm{f})} + \frac{\partial \boldsymbol{P}}{\partial t} + \nabla \times \boldsymbol{M}$$

$\boldsymbol{D}$ and $\boldsymbol{H}$

$$\nabla \cdot \boldsymbol{E} = \frac{1}{\varepsilon_0}\,\rho$$

$$\nabla \times c\boldsymbol{B} - \frac{1}{c}\frac{\partial \boldsymbol{E}}{\partial t} = \frac{1}{\varepsilon_0 c}\,\boldsymbol{j}$$

- Insert previous page's definition and move $\boldsymbol{P}$ and $\boldsymbol{M}$ to the other side
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- We customary use $\boldsymbol{D} = \varepsilon_0 \boldsymbol{E} + \boldsymbol{P}$ and $\boldsymbol{H} = \varepsilon_0 c^2 \boldsymbol{B} - \boldsymbol{M}$ (more units 👎)
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$\boldsymbol{D}$ and $\boldsymbol{H}$

$$\nabla\cdot\boldsymbol{E}=\frac{1}{\varepsilon_0}\left(\rho^{(\mathrm{f})}-\nabla\cdot\boldsymbol{P}\right)$$

$$\nabla\times c\boldsymbol{B}-\frac{1}{c}\frac{\partial\boldsymbol{E}}{\partial t}=\frac{1}{\varepsilon_0 c}\left(\boldsymbol{j}^{(\mathrm{f})}+\frac{\partial\boldsymbol{P}}{\partial t}+\nabla\times\boldsymbol{M}\right)$$

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$\boldsymbol{D}$ and $\boldsymbol{H}$

$$\nabla \cdot \left(\boldsymbol{E} + \boldsymbol{P}/\varepsilon_0 \right) = \frac{1}{\varepsilon_0} \rho^{(\mathrm{f})}$$

$$\nabla \times \left(c\boldsymbol{B} - \boldsymbol{M}/\varepsilon_0 c \right) - \frac{1}{c} \frac{\partial (\boldsymbol{E} + \boldsymbol{P}/\varepsilon_0)}{\partial t} = \frac{1}{\varepsilon_0 c}\, \boldsymbol{j}^{(\mathrm{f})}$$

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$\boldsymbol{D}$ and $\boldsymbol{H}$

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$\boldsymbol{D}$ and $\boldsymbol{H}$

$$\nabla\cdot\left(\boldsymbol{E}+\boldsymbol{P}/\varepsilon_0\right)=\rho^{(\mathrm{f})}/\varepsilon_0$$

$$c^2\nabla\times\left(\boldsymbol{B}-\boldsymbol{M}/\varepsilon_0 c^2\right)-\frac{\partial(\boldsymbol{E}+\boldsymbol{P}/\varepsilon_0)}{\partial t}=\frac{\boldsymbol{j}^{(\mathrm{f})}}{\varepsilon_0}$$

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- Feynman used $\boldsymbol{H}=\boldsymbol{B}-\boldsymbol{M}/\varepsilon_0 c^2$ to make $\boldsymbol{H}$ to have the same units as $\boldsymbol{B}$

Macroscopic Maxwell's equations (SI with ε_0 and c)

$$\nabla\times\boldsymbol{E}+\frac{\partial\boldsymbol{B}}{\partial t}=0 \qquad \nabla\cdot\boldsymbol{B}=0$$

$$\nabla\cdot\boldsymbol{D}=\rho^{(\mathrm{f})} \qquad \boldsymbol{D}=\left(\varepsilon_0\boldsymbol{E}+\boldsymbol{P}\right)$$

$$\nabla\times\boldsymbol{H}-\frac{\partial\boldsymbol{D}}{\partial t}=\boldsymbol{j}^{(\mathrm{f})} \qquad \boldsymbol{H}=\left(\varepsilon_0c^2\boldsymbol{B}-\boldsymbol{M}\right)$$

$\boldsymbol{E}$, $\boldsymbol{B}$, $\boldsymbol{D}$, $\boldsymbol{H}$ are in different units

Some people in the past thought this is cleaner, because constants are hidden

Macroscopic Maxwell's equations (Feynman)

$$\nabla \times \boldsymbol{E} + \frac{\partial \boldsymbol{B}}{\partial t} = 0 \qquad \nabla \cdot \boldsymbol{B} = 0$$

$$\nabla \cdot (\varepsilon_0 \boldsymbol{E} + \boldsymbol{P}) = \rho_{\text{other}} \qquad \boldsymbol{D}/\varepsilon_0 = \left(\boldsymbol{E} + \boldsymbol{P}/\varepsilon_0\right)$$

$$c^2 \nabla \times \left(\boldsymbol{B} - \frac{\boldsymbol{M}}{\varepsilon_0 c^2}\right) = \frac{\boldsymbol{j}_{\text{cond}}}{\varepsilon_0} + \frac{\partial}{\partial t}\left(\boldsymbol{E} - \frac{\boldsymbol{P}}{\varepsilon_0}\right) \qquad \boldsymbol{H} = \left(\boldsymbol{B} - \frac{\boldsymbol{M}}{\varepsilon_0 c^2}\right)$$

Please read *FLP* Vol II Chap 36

Macroscopic Maxwell's equations (Gaussian units)

$$\nabla \times \boldsymbol{E} + \frac{1}{c}\frac{\partial \boldsymbol{B}}{\partial t} = 0 \qquad \nabla \cdot \boldsymbol{B} = 0$$

$$\nabla \cdot \boldsymbol{D} = 4\pi\,\rho^{(\mathrm{f})} \qquad \boldsymbol{D} = (\boldsymbol{E} + 4\pi \boldsymbol{P})$$

$$\nabla \times \boldsymbol{H} - \frac{1}{c}\frac{\partial \boldsymbol{D}}{\partial t} = \frac{4\pi}{c}\,\boldsymbol{j}^{(\mathrm{f})} \qquad \boldsymbol{H} = (\boldsymbol{B} - 4\pi \boldsymbol{M})$$

$\boldsymbol{E}$, $\boldsymbol{B}$, $\boldsymbol{D}$, $\boldsymbol{H}$ are in the same unit

$\boldsymbol{D}$ and $\boldsymbol{H}$ in Gaussian are different quantities than those in SI

Macroscopic Maxwell's equations (Gaussian units with ε_0)

$$\nabla \times \boldsymbol{E} + \frac{1}{c}\frac{\partial \boldsymbol{B}}{\partial t} = 0 \qquad \nabla \cdot \boldsymbol{B} = 0$$

$$\nabla \cdot \boldsymbol{D} = \rho^{(\mathrm{f})}/\varepsilon_0 \qquad \boldsymbol{D} = (\boldsymbol{E} + \boldsymbol{P}/\varepsilon_0)$$

$$\nabla \times \boldsymbol{H} - \frac{1}{c}\frac{\partial \boldsymbol{D}}{\partial t} = \frac{1}{\varepsilon_0 c}\boldsymbol{j}^{(\mathrm{f})} \qquad \boldsymbol{H} = (\boldsymbol{B} - \boldsymbol{M}/\varepsilon_0)$$

Substitute 4π with $1/\varepsilon_0$ to give charge a dimension

Macroscopic Maxwell's equations (Gaussian units with ε_0)

$$\nabla \times \boldsymbol{E} + \frac{1}{c}\frac{\partial \boldsymbol{B}}{\partial t} = 0 \qquad \nabla \cdot \boldsymbol{B} = 0$$

$$\nabla \cdot \boldsymbol{E} = \frac{1}{\varepsilon_0}\left(\rho^{(\mathrm{f})} - \nabla \cdot \boldsymbol{P}\right)$$

$$\nabla \times \boldsymbol{B} - \frac{1}{c}\frac{\partial \boldsymbol{E}}{\partial t} = \frac{1}{\varepsilon_0 c}\left(\boldsymbol{j}^{(\mathrm{f})} + \frac{\partial \boldsymbol{P}}{\partial t} + \nabla \times c\boldsymbol{M}\right)$$

EM field in the left, material in the right hand side

You can go to SI with $\boldsymbol{B} \to c\boldsymbol{B}$ and $\boldsymbol{M} \to \boldsymbol{M}/c$

$c \quad \varepsilon_0 \quad q_e$

"$\sim$" means measured, "$=$" means defined q_e is elementary charge

- Gaussian

$$c \sim 2.998 \times 10^{10}\,\mathrm{cm/s} \qquad \varepsilon_0 = \frac{1}{4\pi} \qquad q_e \sim 4.803 \times 10^{-10}\,\mathrm{statC}\left(\mathrm{cm}^{3/2}\mathrm{g}^{1/2}\mathrm{s}^{-1}\right)$$

- SI before 2019

$$c = 299792458\,\mathrm{m/s} \qquad \varepsilon_0 = \frac{10^7}{4\pi\left(c/(\mathrm{m/s})\right)^2}\,\mathrm{F/m} \qquad q_e \sim 1.602 \times 10^{-19}\,\mathrm{C}$$

- SI after 2019

$$c = 299792458\,\mathrm{m/s} \qquad \varepsilon_0 \sim 8.8854 \times 10^{-12}\,\mathrm{F/m} \qquad q_e = 1.602176634 \times 10^{-19}\,\mathrm{C}$$

Gaussian units cannot be accurate theory any longer, because you can't modify $1/4\pi$

Dimensions for SI quantities ($[\circ]$ reads dimension of $\circ$)

$$[\boldsymbol{E}] = [c\boldsymbol{B}] = \left[\frac{\text{Voltage}}{\text{Length}}\right] \qquad [\rho] = \left[\frac{\text{Charge}}{\text{Length}^3}\right] \qquad [\boldsymbol{j}] = \left[\frac{\text{Current}}{\text{Length}^2}\right] \qquad [\varepsilon_0] = \left[\frac{\text{Cap}}{\text{Length}}\right]$$

$$\left[\nabla\cdot\boldsymbol{E} = \frac{\rho}{\varepsilon_0}\right] = \left[\frac{\text{Voltage}}{\text{Length}^2} = \frac{\text{Charge}}{\text{Cap}\cdot\text{Length}^2}\right] \qquad \left(V = \frac{Q}{C}\right) \qquad \left[\varepsilon_0\,\frac{S}{d}\right] = [\text{Cap}]$$

$$[\text{Time}] = [\text{Res}\cdot\text{Cap}] \qquad (\tau = RC) \qquad [1/\varepsilon_0 c] = [\text{Time}/\text{Cap}] = [\text{Res}]$$

$$\left[\nabla\times c\boldsymbol{B} - \frac{1}{c}\frac{\partial \boldsymbol{E}}{\partial t} = \frac{1}{\varepsilon_0 c}\,\boldsymbol{j}\right] = \left[\frac{\text{Voltage}}{\text{Length}^2} = \text{Res}\cdot\frac{\text{Current}}{\text{Length}^2}\right] \qquad (V = R\,I)$$

$$[\boldsymbol{j} = \sigma\boldsymbol{E}] = \left[\frac{\text{Current}}{\text{Length}^2} = \frac{1}{\text{Res}\cdot\text{Length}}\cdot\frac{\text{Voltage}}{\text{Length}}\right] \qquad \left(I = \frac{V}{R}\right)$$

"The difficulty of science are to a large extent the difficulties of notation, the units, and all the other artificialities which are invented by man, not by nature."

— Richard P. Feynman, *The Feynman Lectures on Physics*

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