Diffusion Current Noise in Semiconductor Devices

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Abstract

A simple derivation of diffusion current noise in semiconductor devices is presented. Diffusion current noise explains shot noise behavior of semiconductor devices such as bipolar junction diode, bipolar junction transistor (BJT) and MOSFET in subthreshold region where diffusion current is the dominant current component.

I. INTRODUCTION

Noise in some semiconductor devices have an interesting property. When such a device is biased to a current I much greater than its saturation current I_s, its noise spectral density reduces by half from the amount one would expect from the thermal noise formula, namely,

$$\left\langle i_N^2 \right\rangle_f = 2qI = 2kTg(I),$$

where i_N, $\langle i_N^2 \rangle_f$, and $g(I)$ is noise current, its spectral density per ordinary frequency and conductance of the device, respectively. At equilibrium, where bias current is zero, it comes back to thermal noise formula[1,2]

$$\left\langle i_N^2 \right\rangle_f = 4qI_s = 4kTg(0).$$

This behavior can be explained by assuming total noise is the sum of two shot noises due to forward going current and reverse going current.[2] And it has been shown that shot noise and thermal noise are identical in semiconductor at equilibrium.[2,3] In this paper, I would like to explain this behavior from the noise of diffusion current of classical gas of carriers.

We consider motion of carriers in a conductor of cross sectional area A, length L, resistance R. We set x-axis along with the conductor length and consider x direction of motion only, just for simplicity. It is well known that resistance R is expressed using collision time τ_c, mass of a carrier m, charge of a carrier q and carrier density n as

$$R = L/A\sigma, \quad \sigma = n\,q\,\mu, \quad \mu = q\,\tau_c/m,$$

where σ and μ is conductivity of the conductor and mobility of the carrier, respectively.

II. RANDOM AGITATION FORCE

At equilibrium, equation of motion for each carrier is

$$m\dot{v}_i \;=\; -\frac{m}{\tau_c}\,v_i \;+\; q\,\mathcal{E}_i,$$

where $\mathcal{E}_i$ represents thermal agitation force and we assume its spectral density is flat (white). When energy loss by τ_c and power provided by $\mathcal{E}_i$ balances each other, the system is at equilibrium. The magnitude of $\mathcal{E}_i$ can be found by this condition as follows. From Laplace transform of the equation

$$v_i(s)/\mathcal{E}_i \;=\; \frac{q\,\tau_c}{m}\cdot\frac{1}{1+s\,\tau_c},$$

we get relation between spectral densities

$$\left\langle v_i^2\right\rangle_\omega = \frac{(q\,\tau_c)^2}{m^2}\cdot\frac{1}{1+\omega^2\tau_c^2}\;\left\langle \mathcal{E}_i^2\right\rangle_\omega, \tag{1}$$

and mean square velocity

$$\left\langle v_i^2\right\rangle = \int_0^\infty \left\langle v_i^2\right\rangle_\omega d\omega = \frac{\pi}{2}\cdot\frac{q^2\tau_c}{m^2}\left\langle \mathcal{E}_i^2\right\rangle_\omega.$$

At equilibrium this should be equal to the thermal velocity kT/m, therefore spectral density of agitation force is found to be

$$\left\langle \mathcal{E}_i^2\right\rangle_\omega = \frac{4kT}{2\pi}\cdot\frac{1}{q\mu} = \frac{4kT}{2\pi}\cdot\frac{n}{\sigma} \tag{2}$$

III. NOISE CURRENT

Recalling that current is expressed by number of carriers inside the conductor N as

$$I = An\,q\,\bar{v} = ALn\,q\,\bar{v}/L = N\bar{v}\,q/L,$$

and that $\bar{v}$ is average of each carrier's velocity:

$$\bar{v} = \frac{1}{N}\sum_i v_i \quad\text{or}\quad \left\langle \bar{v}^2\right\rangle = \frac{1}{N^2}\,N\left\langle v_i^2\right\rangle = \frac{1}{N}\left\langle v_i^2\right\rangle,$$

we see noise current is expressed by mean square of carrier velocity as follows:

$$\begin{aligned}\left\langle i_N^2\right\rangle &= \left\langle (I-\bar{I})^2\right\rangle,\\ &= N^2q^2\left\langle \bar{v}^2\right\rangle/L^2,\\ &= Nq^2\left\langle v_i^2\right\rangle/L^2.\end{aligned}$$

Note that average current $\bar{I}$ is zero at equilibrium. Same thing can be said for spectral densities as well:

$$\left\langle i_N^2\right\rangle_\omega = Nq^2\left\langle v_i^2\right\rangle_\omega/L^2$$

Using Eq. (1) and Eq. (2), we find noise spectral density

$$\left\langle i_N^2\right\rangle_\omega = \frac{N}{L^2}\,q^2\left\langle v_i^2\right\rangle_\omega = \frac{Nq\mu}{L^2}\cdot\frac{4kT}{2\pi}\cdot\frac{1}{1+\omega^2\tau_c^2}. \tag{3}$$

This reduces to the thermal noise formula. For $\omega \ll 1/\tau_c$,

$$\left\langle i_N^2\right\rangle_f = 2\pi\left\langle i_N^2\right\rangle_\omega = 4kT/R.$$

IV. DRIFT CURRENT : RESISTORS

Thermal velocity v_T can be written with mean free path $l = v_T\,\tau_c$ as

$$v_T = v_T^2/v_T = \frac{kT}{m}\cdot\frac{\tau_c}{l} \;=\; \mu\,\frac{kT/q}{l}.$$

Recalling that drift velocity v_D is $\mu\bar{E}=\mu V/L$, the ratio between drift velocity and thermal velocity is

$$\frac{v_D}{v_T} = \frac{V}{kT/q}\cdot\frac{l}{L}.$$

In resistors, mean free path is in the order of nano meters, whereas L is in the order of micro meters, drift current hardly affect each carrier's activity. Therefore noise is not sensitive to drift current.

$$\left\langle v_i^2\right\rangle \;=\; v_D^2 + v_T^2 \;\sim\; v_T^2$$

V. DIFFUSION CURRENT : TRANSISTORS

Suppose that some external force makes non-uniform carrier density. Then, there must be diffusion current

$$j = -qD\,\frac{\partial n(x)}{\partial x}.$$

In case total current is dominated by this diffusion current, gradient of $n(x)$ is constant because current at any cross section must be the same:

$$\frac{\partial n(x)}{\partial x} = \frac{n(L)-n(0)}{L} = \frac{n(0)}{L}\left(\frac{n(L)}{n(0)}-1\right).$$

Therefore total current

$$I = Aj = \frac{A\,qD\,n(0)}{L}\left(1-\frac{n(L)}{n(0)}\right) = I_s\left(1-\frac{n(L)}{n(0)}\right).$$

Depletion region of p-n junction makes it possible to create such situation described here (carrier density gradient, absence of drift current). In subthreshold MOSFET, for example, $n(0)\propto\exp(-qV_{gs}/kT)$ and $n(L)/n(0)=\exp(qV_{ds}/kT)$.[4] Therefore

$$I_d \;\propto\; \frac{A}{L}\,e^{-qV_{gs}/kT}\left(1-e^{qV_{ds}/kT}\right).$$

As for noise, since diffusion current is nothing but current driven by thermal agitation force, we can use Eq. (3). Inserting $N = AL(n(0)+n(L))/2$ into Eq. (3) and using Nernst-Einstein[5] relation $D = \mu kT/q$ yields

$$\left\langle i_N^2 \right\rangle_\omega = \frac{2q}{2\pi} \cdot \frac{A\, qD\, n(0)}{L} \left(1 + \frac{n(L)}{n(0)}\right).$$

This reproduces the shot noise behavior of semiconductor devices. For example, noting that device current I is very close to I_s for $n(0) \gg n(L)$, above is reduced to "shot noise" formula. In ordinary frequency,

$$\left\langle i_N^2 \right\rangle_f = 2qI.$$

And at equilibrium where $n(L) = n(0)$,

$$\left\langle i_N^2 \right\rangle_f = 4qI_s.$$

Appendix A: Electromotive force

For arbitrary cross section, there is nA carriers per unit length and average agitation force is

$$\left\langle \bar{\mathcal{E}}^2 \right\rangle = \frac{1}{(nA)^2} \cdot nA \left\langle \mathcal{E}_i^2 \right\rangle = \frac{1}{nA} \left\langle \mathcal{E}_i^2 \right\rangle.$$

Mean square electromotive force across the conductor can be summed up for the conductor's length L:

$$\left\langle V^2 \right\rangle = L \left\langle \bar{\mathcal{E}}^2 \right\rangle = \frac{L}{nA} \left\langle \mathcal{E}_i^2 \right\rangle.$$

Same thing can be said for each frequency component:

$$\left\langle V^2 \right\rangle_\omega = \frac{L}{nA} \left\langle \mathcal{E}_i^2 \right\rangle_\omega.$$

Therefore

$$\left\langle V^2 \right\rangle_\omega = \frac{L}{nA} \left\langle \mathcal{E}_i^2 \right\rangle_\omega = 4kTR \,/\, 2\pi.$$

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